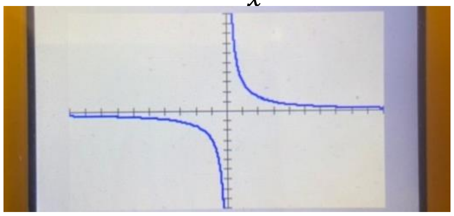
Sophomore and Junior Assignments- Becker

Graphing Rational Functions

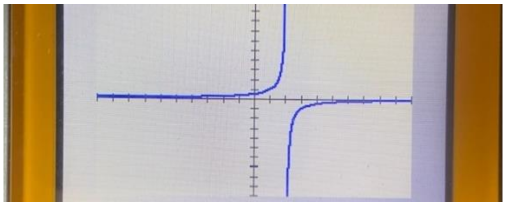
- What is a rational function?
  - A rational function is a function expressed as a **ratio**, or **fraction**.
  - For example; if we are given $xy = 3$ we can rearrange it to equal $y = \frac{3}{x}$ by dividing the variable x on both sides. This is called $\boldsymbol{INVERSE\ VARIATION}$ since we inverted the equation with the manipulation. We can ***GRAPH*** this inverse relation. It will **NOT** create a straight line *when you have a variable in the denominator.* It creates a fraction that changes as you change the value of x, if you plotted each value of x and the fraction it creates, it would create a curved line when you connect the dots.
    - $(1,3); \left(2,\frac{3}{2}\right); (3,1); \left(4,\frac{3}{4}\right); \left(5,\frac{3}{5}\right); \left(6,\frac{1}{2}\right); \left(7,\frac{3}{7}\right); \left(8,\frac{3}{8}\right); (9,\frac{1}{3})$
    - Note the $y\ coordinate$; as you plug in larger values of x in the denominator the number gets smaller. *All fractions are reduced.*
    - $The\ same\ would\ be\ true\ for\ all\ negative\ value\ of\ x\ also$
    - $With\ both\ positive\ and\ negative\ values\ being\ graphed, the\ graph\ is$:

$$y = \frac{3}{x}$$



  - *Remember:* it is NOT possible to divide by the number 0 in ANY fraction. So wherever the denominator equals 0, we have what is called an $\boldsymbol{ASYMPTOTE}$. An asymptote is a vertical line through the graph at the value where the $denominator\ equals\ zero$, in this case that line is at $x = 0$
  - In the example above; $y = \frac{3}{x}$; if we take the denominator and set it equal to zero and solve the equation, we will find the $x\ coordinate\ value$ of where the graph will have a vertical line. You can see in this example there is a line down the middle of the graph that both sides approach, but never touch

  - If we look at a different example; $y = \frac{1}{(x-2)}$ and solve where the denominator equals zero: $x - 2 = 0; x - 2 + \mathbf{2} = 0 + \mathbf{2}\ becomes\ x = 2$. That means there will be a vertical line at the value of $x = 2$. The graph is shown below. Notice the vertical line at $x = 2$ and that the line approaches but never touches that value. You could create a table of $x\ and\ y\ values$ to find each specific coordinate if you were asked by *evaluating the expression.*
    - The negative in the denominator causes a shift in the asymptote, with the asymptote at the value of $x = 2$. The horizontal asymptote is represented by $y = 0$



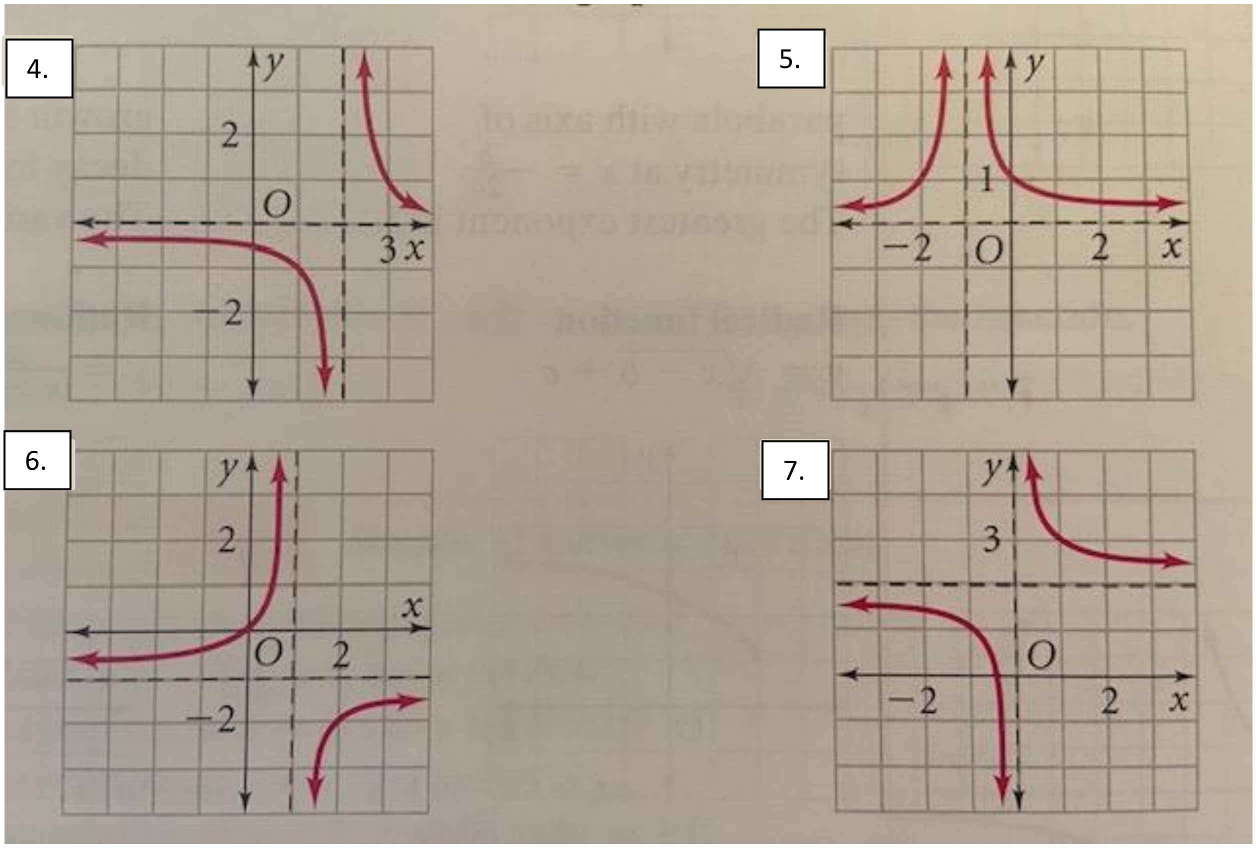
Identify the asymptotes of each equation by solving where the denominator of each function equals zero.

1. $f(x) = \frac{2}{x}$

2. $y = \frac{1}{x+2}$

3. $h(x) = \frac{3}{2x-4}$

<u>Identify the asymptotes of each graph on both x and y axis.</u> For the vertical asymptote, write $x = your\ answer$. For the horizontal asymptote, write $y = you\ answer$. Remember, a horizontal line is graphed $y = a$ while a vertical line is graphed $x = a$, where a is the value where the line is.



<u>Simplifying rational expressions</u>

- What is a rational expression? A rational expression is an expression of two polynomials in a ratio, or a fraction. For example; $\frac{6x+12}{x+2}$ is a rational expression. Both numerator and denominator are polynomials.
- However, all this tells us is we have a *<u>vertical asymptote</u>* at the value of $x = -2$ if we were asked to graph. We want to simplify it down if possible. If we factor a 6 out of the numerator the equation becomes

  $\frac{6(x+2)}{x+2}$ now we have the number of $(x + 2)$ in both numerator and denominator that we can cancel

$$\frac{6\cancel{(x+2)}}{\cancel{x+2}} = \frac{6}{1} = 6$$

- The same rule applies if there is a quadratic in the numerator or denominator. We must first factor the quadratic then find numbers that cancel out in order to $simplify$.

  $\frac{2x-12}{x^2-7x+6}$ here we must first factor both top and bottom, then simplify.

  *FOR A REVIEW ON FACTORING, ASK FOR ADDITIONAL LESSON

$$\frac{2x-12}{x^2-7x+6} = \frac{2(x-6)}{(x-6)(x-1)} = \frac{2\cancel{(x-6)}}{\cancel{(x-6)}(x-1)} = \frac{2}{(x-1)}$$

Simplify each rational expression:

8. $\frac{6a+9}{12}$

9. $\frac{2m-5}{6m-15}$

10. $\frac{3x^2-9x}{x-3}$

11. $\frac{2b-8}{b^2-16}$

12. $\frac{m^2+7m+12}{m^2+6m+8}$

__Adding/Subtracting Rational expressions__

In order to add any fraction, we must have a *common denominator.* If the denominators are the same, we can add our fraction straight across. This is also true of a fraction with a variable in the denominator.

$$\frac{5}{2m}+\frac{4}{2m}=\frac{9}{2m}$$

$$\frac{3n+4}{2n^2+5n-3}-\frac{2n+1}{2n^2+5n-3}=\frac{3n+4-(2n+1)}{2n^2+5n-3}=\frac{3n+4-2n-1}{2n^2+5n-3}=\frac{n+3}{2n^2+5n-3}$$

Now, factor the quadratic on the bottom using the *factoring by grouping* method since we have a coefficient that is not the value of 1.

$$\frac{n+3}{(2n-1)(n+3)}=\frac{\cancel{n+3}}{(2n-1)\cancel{(n+3)}}=\frac{1}{2n-1}$$

However, if the denominator is not the same we must first make it the same for both fractions before we can add.

$$\frac{2}{3x}+\frac{1}{6}.$$

Find the lease common denominator, which will be $6x$. In order to get that in both fractions, we must multiply the first fraction by 2 and the second fraction by x. $\frac{2}{3x}+\frac{1}{6}=\frac{2*2}{2*3x}+\frac{1*x}{6*x}=\frac{4}{6x}+\frac{x}{6x}$. Now we can add or subtract our fraction straight across.

$$\frac{4}{6x}+\frac{x}{6x}=\frac{4+x}{6x}; we\ cannot\ reduce\ any\ further$$

The same is true for more complicated denominators. Multiply the denominator to both top and bottom of the other fraction and it will give you a common denominator that allows you to add the fraction.

$$\frac{5}{c+2}+\frac{6}{c-3}=$$

$$\frac{5(c-3)}{(c-3)(c+2)}+\frac{6(c+2)}{(c-3)(c+2)}=\frac{5c-15}{(c+2)(c-3)}+\frac{6c+12}{(c+2)(c-3)}=\frac{11c-3}{(c+2)(c-3)}$$

ADD OR SUBTRACT

If a common denominator exists, add straight across. If a common denominator does not exist, use multiplication to find one.

13. $\frac{4}{6t-1}+\frac{3}{6t-1}$

14. $\frac{n}{n+3}+\frac{2}{n+3}$

15. $\frac{3y+2}{y+4}-\frac{y-6}{y+4}$

16. $\frac{7}{3a}+\frac{2}{5}$

17. $\frac{3}{8m^3} + \frac{1}{12m^2}$

18. $\frac{c}{c+5} + \frac{4}{c+3}$

19. $\frac{c^2}{ab} - \frac{a^2}{bc}$

20. $9 + \frac{x-3}{x+2}$